TURBO-ICT PICO-COULOMB CALIBRATION TO PERCENT-LEVEL ACCURACY

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Abstract

We report on the calibration methods implemented for the Turbo-ICT and the BCM-RF. They allow to achieve percent-level accuracy for charge and current measurements. Starting from the Turbo-ICT and BCM-RF working principle, we discuss the scientific fundaments of their calibration and the practical implementation in a test bench. Limits, both principle and practical, are reviewed. Achievable accuracy is estimated.

INTRODUCTION

The Turbo-ICT sensor and its corresponding BCM-RF electronics can accurately measure charges of ultra-short particle bunches as well as average currents of CW beams of such bunches [1,2].

When excited by a single bunch, the Turbo-ICT output signal is a short resonance at a fixed frequency f_{res} but charge-proportional amplitude. The BCM-RF works in sample-and-hold mode and measures the apex of this resonance. The maximum possible bunch repetition rate is approximately 2 MHz. For calibration the relation between Turbo-ICT input charge Q_{in} and BCM-RF output voltage U_{BCMRF} is determined.

When excited by a CW beam, the Turbo-ICT output signal is a sine wave of frequency f_{res} and current-proportional amplitude. The BCM-RF works in track-continuous mode and measures the apex of this sine wave. The Turbo-ICT resonance frequency f_{res} must match the bunch repetition rate f_{rep} or a harmonic. For calibration the relation between average input current $\langle I_{\mathrm{cw,in}} \rangle$ and BCM-RF output voltage U_{BCMRF} is determined.

In the following, we discuss the Turbo-ICT and BCM-RF working principle. The calibration methods for both modes of operation are described and the achievable accuracies are estimated.

TURBO-ICT / BCM-RF PRINCIPLE

To determine charge or current the BCM-RF measures on a logarithmic scale the apex of the Turbo-ICT output signal. Hence, the apex should depend only on input charge or current. Most notably, any current transformer's output pulse shape is usually dependent on input pulse shape, which could induce a variation of the apex even for constant charge or current. Only for "sufficiently short" input pulses this dependence is negligible.

It is required that an input pulse must be considerably shorter than the Turbo-ICT resonance wave length, which is fulfilled, e.g., in laser-plasma accelerators or X-ray free-electron lasers. Details are given in Appendix A.

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Turbo-ICT Pulse Response

The spectral response $Q_{\mathrm{out}}(f)$ of a Turbo-ICT to an incoming current pulse $I_{\mathrm{in}}(t)$ is the product of the incoming pulse's spectrum $Q_{\mathrm{in}}(f)$ and the Turbo-ICT's transmission coefficient $S_{21}(f)$, e.g. as obtained from S-parameter measurements using a vector network analyser (VNA):

$$Q_{\mathrm{out}}(f) = Q_{\mathrm{in}}(f)\, S_{21}(f)\,.$$

Using the inverse Fourier transform the time-domain output current pulse $I_{\mathrm{out}}(t)$ can be determined:

$$I_{\mathrm{out}}(t) = \int_{-\infty}^{+\infty} Q_{\mathrm{in}}(f)\, S_{21}(f)\, e^{i\,2\pi\,f t}\, df\,.$$

For "sufficiently short" input pulses, $I_{\mathrm{out}}(t)$ can be approximated:

$$I_{\mathrm{out}}(t) \approx Q_{\mathrm{in}} \int_{-\infty}^{+\infty} S_{21}(f)\, e^{i\,2\pi\,f t}\, df = Q_{\mathrm{in}}\, M(t)\,. \quad (1)$$

That means, for "sufficiently short" input pulses the Turbo-ICT output pulse has always the same shape $M(t)$ scaled by the input pulse charge Q_{in}.

$M(t) = \int_{-\infty}^{+\infty} S_{21}(f)\, e^{i\,2\pi\,f t}\, df$ is the Turbo-ICT's response to a Dirac pulse, i.e. to an infinitely short current pulse, normalized by the pulse's charge; its units are Ampère per Coulomb. Figure 1 shows a typical $S_{21}(f)$ of a Turbo-ICT and the corresponding Dirac response.

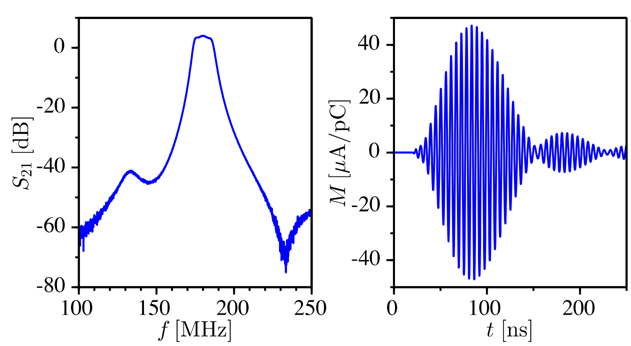

Figure 1: Typical Turbo-ICT response in frequency-domain (left) and in time-domain (right).

Turbo-ICT Dirac Response Correction

As mentioned above, the Turbo-ICT Dirac response can be reconstructed from the Turbo-ICT's S_{21}:

$$M(t) = \int_{-\infty}^{+\infty} S_{21}(f)\, e^{i\,2\pi\,f t}\, df\,.$$

While this equation is in theory correct, it is in practice not sufficient. Around the Turbo-ICT, the measurement setup is not perfectly matched to 50 Ω wave impedance. Reflections occur during the VNA measurements, lowering power and current passing the Turbo-ICT. Such effects will not be present in the accelerator. Consequently, the measured transmission coefficient $S_{21,\mathrm{VNA}}$ is not exactly representative of the real $S_{21,\mathrm{ACC}}$ in the accelerator.

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The required correction can be obtained by exploiting the fact that the ratio of Turbo-ICT output voltage $U_{\mathrm{out,VNA}}$ and input current I_{in} is constant. I_{in} is the current passing the Turbo-ICT, not the current I_0 sent by the source. If I_0 would enter the Turbo-ICT, as in the accelerator due to absence of reflections, the output voltage would be $U_{\mathrm{out,ACC}}$. But the ratio must remain the same:

$$\frac{U_{\mathrm{out,ACC}}}{I_0} = \frac{U_{\mathrm{out,VNA}}}{I_{\mathrm{in}}} = \frac{U_0\, S_{21,\mathrm{VNA}}}{I_{\mathrm{in}}}$$

$$\Leftrightarrow \frac{U_{\mathrm{out,ACC}}}{U_0} = S_{21,\mathrm{VNA}} \frac{I_0}{I_{\mathrm{in}}} = S_{21,\mathrm{ACC}} \,.$$

The input current I_{in} can be calculated either using the reflection coefficient $S_{11,\mathrm{VNA}}$, i.e. the signal reflected at the Turbo-ICT input, or using the transmission coefficient $S_{31,\mathrm{VNA}}$, i.e. the signal passing the Turbo-ICT:

$$I_{\mathrm{in}} = I_0\left(1 - S_{11,\mathrm{VNA}}\right) = I_0\, S_{31,\mathrm{VNA}}$$

and we get:

$$S_{21,\mathrm{ACC}} = \frac{S_{21,\mathrm{VNA}}}{1 - S_{11,\mathrm{VNA}}} = \frac{S_{21,\mathrm{VNA}}}{S_{31,\mathrm{VNA}}}\,.$$

In practice it is less error-prone to use $S_{31,\mathrm{VNA}}$. Its phase has an impact only on the phase of $S_{21,\mathrm{ACC}}$, but not on the absolute value. However, to obtain $S_{31,\mathrm{VNA}}$ a 3-port S-parameter measurement is required.

The correct $M(t)$ that must be used for calculations is:

$$M_{\mathrm{correct}}(t) = \int_{-\infty}^{+\infty} S_{21,\mathrm{ACC}}(f)\, e^{i\,2\pi\, f t}\, df\,.$$

Charge Measurements

The results obtained can be directly exploited for single-bunch charge measurements. Rearranging Eqn. (1) gives:

$$Q_{\mathrm{in}} \approx I_{\mathrm{out}}(t)/M(t)\,.$$

Since Q_{in} is time independent it is, e.g., sufficient to divide the apex of $I_{\mathrm{out}}(t)$ by the apex of $M(t)$ or to divide their respective peak-to-peak values:

$$Q_{\mathrm{in}} \approx \frac{\max(|I_{\mathrm{out}}(t)|)}{\max(|M(t)|)} \approx \frac{\max\left(I_{\mathrm{out}}(t)\right) - \min\left(I_{\mathrm{out}}(t)\right)}{\max\left(M(t)\right) - \min\left(M(t)\right)}\,. \qquad (2)$$

Current Measurements

The spectrum of a CW beam of equal bunches consists only of a DC component, a component at the bunch repetition frequency f_{rep} and its harmonics.

Since the Turbo-ICT includes a narrow band-pass filter around $f_{\mathrm{res}} = f_{\mathrm{rep}}$, or a harmonic of f_{rep}, only a single frequency is transmitted. That means, for a CW input beam the Turbo-ICT output signal is a sine wave. This can also be understood by considering that in time-domain the output signal must be the sum of the time-shifted resonances excited by consecutive bunches.

The output amplitude $I_{\mathrm{rms,out}}$ can be related to the average input current $\langle I_{\mathrm{cw,in}}\rangle$ (see Appendix B):

$$\langle I_{\mathrm{cw,in}}\rangle \approx \frac{I_{\mathrm{rms,out}}}{\sqrt{2}\, S_{21,\mathrm{ACC}}\left(f_{\mathrm{rep}}\right)}\,. \qquad (3)$$

SINGLE-BUNCH CHARGE CALIBRATION

For single-bunch charge calibration, the Turbo-ICT needs to be excited by a short current pulse, whose charge needs to be determined, to obtain a resonance which can be measured by the BCM-RF.

Equivalent Input Charge

It is important to understand that one has to determine the charge as seen by the Turbo-ICT, which will be excited only by the spectral power falling into its bandwidth.

As mentioned before, for pulse length independent measurements the input pulses need to be "sufficiently short". The pulses generated by our fast pulser, a CPS/1S by Kentech Instruments Ltd., have a FWHM length of 200ps, which could suffice in some cases. But in practice they are too long due to having a tail. Additionally, cable losses stretch the pulses further.

To circumvent this problem, an equivalent input charge is determined from the Turbo-ICT output resonance. The Turbo-ICT is excited using the fast pulser and a programmable step attenuator. The resonance peak-to-peak value is measured by an oscilloscope. The Turbo-ICT Dirac response is reconstructed from S-parameters. Eqn. (2) is applied to obtain the equivalent input charge, which is the charge of a Dirac pulse that would excite the same resonance as the fast pulser.

Charge Scan

To obtain the BCM-RF response, a charge scan is performed using the fast pulser and the programmable step attenuator. For each attenuator setting the BCM-RF output voltage U_{BCMRF} is recorded. Based on the attenuator settings and the previously determined equivalent input charge, output voltage and input charge are related.

Estimation of Calibration Accuracy

While the calibration principle is rather straight forward, there are a few issues that need to be considered to achieve good calibration accuracy. Some are also important for the measurements in the accelerator.

First, the accuracy of the equivalent input charge determination depends on the accuracy of the Turbo-ICT resonance as measured by the oscilloscope and the accuracy of the Turbo-ICT Dirac response reconstruction from VNA measurements. These two points are further discussed in the following sub-sections.

Second, the BCM-RF output voltage is a DC voltage which can be easily measured with sufficient precision. Hence, it has no impact on aggregate accuracy.

Third, the BCM-RF must properly measure the resonance apex. To achieve this, the sample-and-hold trigger has to be finely adjusted. Only if the trigger is set up correctly it does not impact accuracy.

Fourth, cable losses must be measured and signal amplitudes need to be corrected accordingly. For calibration, losses in the cable connecting Turbo-ICT and BCM-RF can be accurately measured using a VNA. In the accelera-

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tor, on the other hand, it might be more difficult to measure these losses. When estimating the calibration accuracy a typical error of 1% is included due to cables.

Fifth, since Turbo-ICT and BCM-RF work at a narrow frequency band their adaptation to 50 Ω is very good. Standing waves on the cable connecting them are negligible. When using the same cable for calibration and in the accelerator, standing waves would in any case be correctly taken into account.

Sixth, during analysis of the charge scan the equivalent input charge needs to be scaled by the real attenuation of the programmable step attenuator, which is measured using a VNA. But also the determination of the equivalent input charge depends on this attenuator. Hence, any systematic scaling error of its real attenuation is compensated. Remaining errors can be neglected.

Accuracy of Resonance Measurements

The Turbo-ICT output resonance is characterized by measuring its peak-to-peak value with an oscilloscope. Noise is reduced by averaging. However, a scaling error might be present due to oscilloscope errors and incomplete knowledge of cable losses.

An improvement is to compare on the oscilloscope the Turbo-ICT resonance to a sine wave of same amplitude and frequency f_{res}. The sine wave is generated by a calibrated RF signal generator. Its peak-to-peak value is deduced from the RF signal generator power setting. By doing so, oscilloscope scaling errors are replaced by RF signal generator errors, which are usually smaller. Fewer cables need to be taken into account.

For Turbo-ICT calibration an Agilent N5181A RF signal generator is used. Its calibration report states an uncertainty of 0.2dB, i.e. about 2%. To remain conservative, a measurement error of 3% is assumed.

Accuracy of Dirac Response Reconstruction

The accuracy of the Dirac response reconstruction is given by the accuracy of the S-parameter measurements. These are performed using a factory calibrated Agilent E5071C 4-port vector network analyser. To correct for the influence of cables, an on-site calibration is performed using a factory calibrated Agilent 85033E calibration kit.

According to data sheets and calibration certificates the absolute accuracies of S_{21} and S_{31} measurements are of the order of 1% amplitude and 1° phase.

Since the ratio S_{21}/S_{31} is used, correlated errors will be eliminated. Uncorrelated errors will increase. It is assumed that the real error is a mixture of correlated and uncorrelated errors and that the error on the ratio will be similar to the error of a single measurement.

In addition, a coaxial structure is required geometrically adapting the cables to the Turbo-ICT aperture. The previously described S_{21} correction only corrects the error due to an impedance mismatch at this structure's input. If the wave impedance along the Turbo-ICT differs from this input impedance, an uncorrected error remains.

When testing the impact of adapting to different wave impedances, a variation of the S_{21} amplitude by 1-2% was observed. For calibration, the setup has been adapted to match a theoretical wave impedance of 50 Ω.

Taking into account these measurement setup uncertainties, an estimated error on the Dirac response amplitude of 2% seems to be justified.

Resulting Accuracy of Charge Calculation

Since above mentioned errors are systematic errors, the worst case would be if they all go in the same direction. In such a case, the errors of resonance measurement, Dirac response reconstruction and cable losses simply add:

$$\Delta_{\mathrm{charge,worst}} \approx 3\% + 2\% + 1\% \approx 6\%\,.$$

However, the errors are independent and the more realistic statistical error is

$$\Delta_{\mathrm{charge}} \approx \sqrt{(3\%)^2 + (2\%)^2 + (1\%)^2} \approx 4\%\,.$$

CW BEAM CURRENT CALIBRATION

Following from the relation between the CW beam's average input current $\langle I_{\mathrm{cw,in}}\rangle$ and the output sine wave's RMS amplitude $I_{\mathrm{rms,out}}$ (see Appendix B), calibration in track-continuous mode can be simplified by using a sine wave as input signal. Taking into account the required correction of the measured S-parameters, we get:

$$\langle I_{\mathrm{cw,in}}\rangle \approx \frac{I_{\mathrm{rms,out}}}{\sqrt{2}\,S_{21,\mathrm{ACC}}(f_{\mathrm{rep}})} = \frac{I_{\mathrm{rms,in}}}{\sqrt{2}}\,\frac{S_{21,\mathrm{VNA}}(f_{\mathrm{rep}})}{S_{21,\mathrm{ACC}}(f_{\mathrm{rep}})}\,.$$

Using a calibrated RF signal generator, the Turbo-ICT is excited by a sine wave of frequency f_{res} and known RMS amplitude. The BCM-RF measures the apex of the Turbo-ICT output sine wave. By applying above equation the average input current of a CW beam is deduced which would lead to the same BCM-RF output voltage.

Estimation of Calibration Accuracy

As for the single-bunch charge calibration, several effects need to be considered to obtain good accuracy.

The following assumptions seem justified. The BCM-RF output voltage is considered error free. A typical error of 1% is included due to cables. The Turbo-ICT Dirac response is known to 2%. And the RF signal generator amplitude accuracy is 2%.

Resulting Accuracy of Current Calculation

As for single-bunch charge calibration, the worst case would be if all errors add:

$$\Delta_{\mathrm{current,worst}} \approx 2\% + 2\% + 1\% \approx 5\%\,.$$

The more realistic statistical error is

$$\Delta_{\mathrm{current}} \approx \sqrt{(2\%)^2 + (2\%)^2 + (1\%)^2} \approx 3\%\,.$$

CONCLUSION

Turbo-ICT and BCM-RF can accurately measure single-bunch charges and CW beam average currents.

Their calibration is derived from a combination of time-domain and frequency-domain measurements. Standard techniques and mathematics are exploited. Based on the accuracy of the instruments used and considering measurement setup uncertainties, absolute calibration errors of

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$\Delta_{\text{charge}} \approx 4\%$ (single-bunch charge)
$\Delta_{\text{current}} \approx 3\%$ (CW beam average current)
are estimated.

To achieve correct measurement results during calibration and in the accelerator, the particle bunch length has to fulfil $t_{\text{FWHM,in}} \lesssim 0.05/f_{\text{res}}$. The bunch should not have a tail.

ACKNOWLEDGMENT

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APPENDIX A SHORT-PULSE ASSUMPTION

The Fourier transform of a finite current pulse $I_{\text{in}}(t)$ is:

$$Q_{\text{in}}(f) = \int_{-\infty}^{+\infty} I_{\text{in}}(t)\, e^{-i\,2\pi\,f t}\, dt$$
$$= \int_{0}^{t_{max}} I_{\text{in}}(t)\, \big(\cos(2\pi\,f t) + i\,\sin(2\pi\,f t)\big)\, dt\,.$$

t_{max} is the total pulse length. If $f \ll 1/2\pi t_{\text{max}}$ the sine approaches zero while the cosine approaches unity for any time t within the integration boundaries. That means, irrespective of the shape of $I_{\text{in}}(t)$ its spectrum $Q_{\text{in}}(f)$ must approach towards DC the value $\int_{-\infty}^{+\infty} I_{\text{in}}(t)\, dt$, i.e. the pulse charge Q_{in}. The spectral amplitude $Q_{\text{in}}(0)$ always exactly equals the pulse charge Q_{in}.

The smaller t_{max} the higher will be the frequencies for which $Q_{\text{in}}(f)$ can be approximated by Q_{in}. In case of the Turbo-ICT, an input pulse can be considered "sufficiently short" only if $Q_{\text{in}}(f) \approx Q_{\text{in}}$ over the full Turbo-ICT bandwidth $S_{21}(f)$.

Assuming a Gaussian input pulse, the maximum length $\sigma_{\text{in,max}}$ can be calculated for which $Q(f_{\text{res}})$, i.e. the spectral amplitude at the Turbo-ICT resonance frequency, lies within a certain fraction ε of Q_{in}:

$$(1-\varepsilon)\, Q_{\text{in}} < Q_{\text{in}}\, e^{-2\pi^2\, \sigma_{\text{in,max}}^2\, f_{\text{res}}{}^2}$$
$$\Leftrightarrow \sigma_{\text{in,max}} < \frac{1}{f_{\text{res}}} \sqrt{\frac{\log(1-\varepsilon)}{-2\pi^2}}\,.$$

If the spectral amplitude $Q(f_{\text{res}})$ should stay within 1% of Q_{in}, the input pulse length needs to fulfil

$$\sigma_{\text{in,max}} < 0.0226/f_{\text{res}}\,.$$

Pulse shapes other than Gaussians will lead to different, though comparable results. Considering only pulses that do not have any tail, we can generally assume that the FWHM of the input pulses should fulfil

$$t_{\text{FWHM,in}} < 0.05/f_{\text{res}}$$

for less than 1% error.

The same limit applies when measuring CW beams. In this case, f_{res} needs to be a harmonic of the pulse repetition rate f_{rep}. The higher the chosen harmonic the tighter is the limit imposed on the input pulse length.

Typically the Turbo-ICT resonance frequency is of the order of 200 MHz, while the spectra of sub-picosecond particle bunches, e.g. generated by laser-plasma accelerators or X-ray free-electron lasers, can reach beyond THz. Such particle bunches can be considered "sufficiently short".

APPENDIX B TURBO-ICT RESPONSE TO CW BEAM

A CW beam of short and equal particle bunches can be mathematically approximated by a Dirac Comb:

$$I_{\text{cw,in}}(t) \approx \sum_{n=-\infty}^{+\infty} Q_{\text{b}}\, \delta(t - n\,T)\,.$$

Q_{b} is the single bunch charge. $T = 1/f_{\text{rep}}$ is the bunch repetition period. The Dirac Comb can be expressed as a Fourier Series:

$$I_{\text{cw,in}}(t) \approx Q_{\text{b}}\, f_{\text{rep}} \sum_{n=-\infty}^{+\infty} e^{i\,2\pi\,n\,f_{\text{rep}}\,t}$$
$$\approx Q_{\text{b}}\, f_{\text{rep}} + 2Q_{\text{b}}\, f_{\text{rep}} \sum_{n=1}^{+\infty} \cos\big(2\pi\,n\,f_{\text{rep}} t\big)\,.$$

$Q_{\text{b}}\, f_{\text{rep}}$ is the average beam current $\langle I_{\text{cw,in}}\rangle$. It corresponds to a DC component in the beam spectrum, which is lost during measurements because current transformers cannot transmit DC components. All other components are scaled by the current transformer's $S_{21}(f)$:

$$I_{\text{out}}(t) \approx 2\, Q_{\text{b}}\, f_{\text{rep}} \sum_{n=1}^{+\infty} \mathrm{S}_{21}\big(n\, f_{\text{rep}}\big) \cos\big(2\pi\,n\,f_{\text{rep}}\,t\big)\,.$$

By band-pass filtering at a single frequency $n\, f_{\text{rep}}$ a cosine signal remains:

$$I_{\text{out,filter}}(t) \approx 2\, Q_{\text{b}}\, f_{\text{rep}}\, \mathrm{S}_{21}\big(n\, f_{\text{rep}}\big)\, \cos\big(2\pi\,n\,f_{\text{rep}}\,t\big)\,.$$

The RMS amplitude of this signal is:

$$I_{\text{out,RMS}} \approx \sqrt{2}\, Q_{\text{b}}\, f_{\text{rep}}\, \mathrm{S}_{21}\big(n\, f_{\text{rep}}\big)$$
$$\Leftrightarrow \langle I_{\text{cw,in}}\rangle = Q_{\text{b}}\, f_{\text{rep}} \approx \frac{I_{\text{out,RMS}}}{\sqrt{2}\, \mathrm{S}_{21}\big(n\, f_{\text{rep}}\big)}\,.$$

An input sine wave of amplitude

$$I_{\text{in,RMS}} = \frac{I_{\text{out,RMS}}}{\mathrm{S}_{21}\big(n\, f_{\text{rep}}\big)}$$

would excite the same Turbo-ICT output signal as the Dirac Comb. This is a consequence of the fact that both input signals deliver the same spectral power at $n\, f_{\text{rep}}$, despite having totally different shapes.

REFERENCES

[1] H.J. Choi, M.S. Chae, H.-S. Kang, and S.J. Park, "Introduction to the Test Results of Turbo-ICT in PAL-ITF", in *Proc. IBIC'14*, Monterey, CA, USA, Sep. 2014, paper WEPF09, pp. 553-557.

[2] K. Nakamura *et al.*, "Pico-coulomb charge measured at BELLA to percent-level precision using a Turbo-ICT", to be published in Plasma Phys. Control. Fusion, LPAW2015 Cluster Issue.

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